# Precision Electro-Weak Parameters from AdS-CFT

hep-ph/0608241 (40 pp.)

hep-ph/0609104 (4 pp.)

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# …in three Acts.

- Introductory hand-waving.
- A well defined model and a detailed calculation.
- Interpreting the result.

# Conclusions

- AdS-CFT correspondence useful tool.
- Non-perturbative effects are (=can be) huge.
- Walking-TC compatible with data.
- Experimental bounds are Nc-independent.
- Spin-1 resonances at 2 TeV.
- Degenerate spectrum of spin-1 states.
- No light scalar (=very broad Higgs at 1-2 TeV?)

- Model building to be done. Can be done.
- LHC phenomenology to be studied. Can be studied.

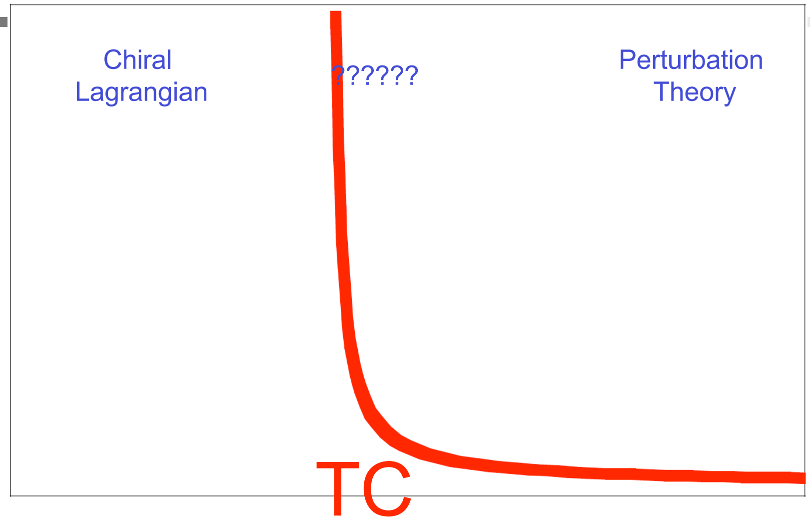
# A Dead Horse

- After LEP, SLAC and Tevatron, Technicolor (naif version of) dismissed, because it does too much:

- S too big.
- T too big.
- Top mass too small.
- Too many PNGB's.
- Too much FCNC.
- Incomprehensible CKM.

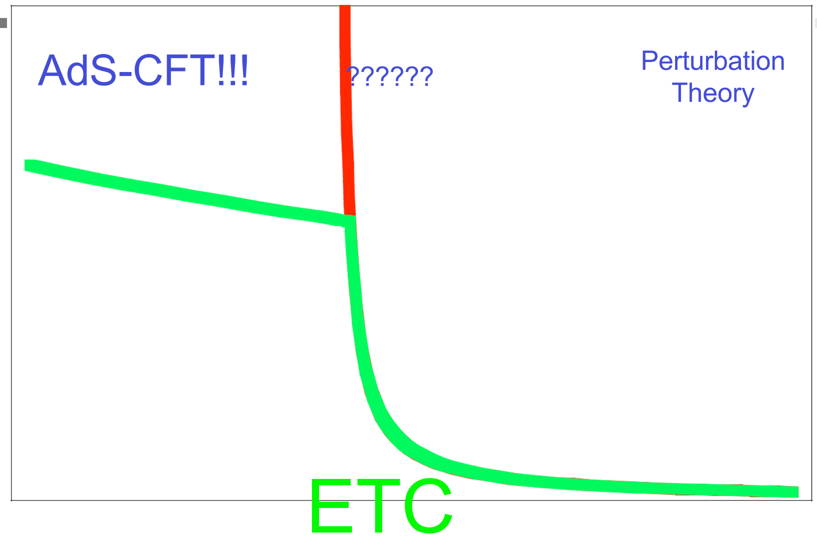
- Too difficult to compute something.
- Too difficult to build a model.

# The “Why”



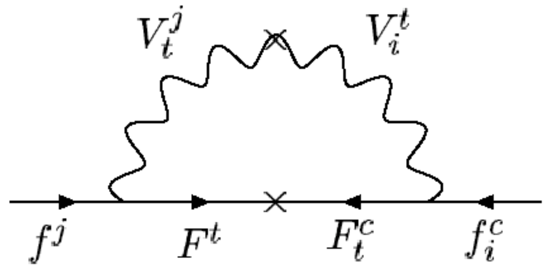

- ONE dynamical scale TC~EWSB.
- Higher-Order operators unsuppressed at electro-weak scale (Little Hierarchy, S, T, FCNC…)
- Computational nightmare at electroweak scale
- Only good: NO big hierarchy problem (conformal symmetry at weak coupling)

# The Solution




- TWO (maybe more…) dynamical scales ETC>>TC~EWSB.
- Higher-Order operators suppressed by large scale (S, T, FCNC…)
- Conformal Symmetry below ETC: little hierarchy solved!
- Computational nightmare at ETC scale ~5-10 TeV: BUT who cares!!!!
- Conformal Symmetry at Large Coupling: Large anomalous dimensions, a new computational tool is need. AdS-CFT!!!!!

# The Top Mass




$$M_{\rm top} \sim \left(\frac{g}{\sqrt{2}}\right)^2 \eta \, \frac{\langle Q^{tT}\, C\, U_t^c \rangle}{M_{ETC}^2}$$

$$= \frac{8\,\pi}{3\,a^2}\,\frac{\Lambda_F^3}{\Lambda_{ETC}^2}\,\eta \quad \lesssim \frac{8\,\pi}{3}\,\frac{\Lambda_F^2}{\Lambda_{ETC}}$$

- If the chiral condensate has dimension d=3, the top mass is ways too small.
- In a CFT at large coupling, there is no reason to think the anomalous dimensions be perturbative. d<3 reasonable.
- For d<3 top mass parametrically enhanced. If d=2 and ETC~4-5 TeV, estimates not parametrically small (maybe topcolor ?) .

# Precision Parameters

- Defined in terms of the polarizations:

$$\hat{S} \equiv \frac{g_4}{g_4'} \pi'_{WB}(0)\,,$$

$$\mathcal{L} = \frac{P_{\mu\nu}}{2} A_i^\mu \pi_{ij}(q^2) A_j^\nu + g_4^a J_{a\mu} A_a^\mu$$

$$\hat{T} \equiv \frac{1}{M_W^2} \left(\pi_{WW}(0) - \pi_+(0)\right)\,,$$

- Tight Experimental Constraints (mH~800 GeV?):

$$\hat{S}_{exp} = (-0.9 \pm 3.9) \times 10^{-3}\,,$$

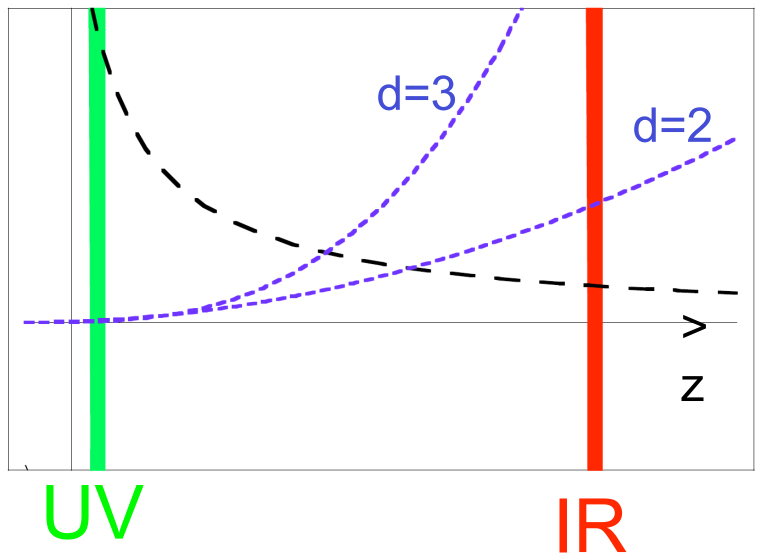
$$\hat{T}_{exp} = (2.0 \pm 3.0) \times 10^{-3}\,,$$

- Custodial Symmetry.
- No Non-Perturbative Estimate for S (as of July 2006).
- Perturbative Estimates are BIG (unless Nc Nd < 8)

$$\hat{S}_p = \frac{\alpha}{4 \sin^2 \theta_W} \frac{N_c N_d}{6\pi}$$

- …but why should we trust this?
- …what is the error?

# The model: geometry




- Gravity Background (AdS5):

$$ds^2 = \left(\frac{L}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

- Boundaries: $L_0 < z < L_1$
- Consistency: $L_0 > L$

# The Model: Action

$$\mathcal{S}_5 \;=\; \int \mathrm{d}^4x \int_{L_0}^{L_1} \mathrm{d}z\, \sqrt{G} \Big[\; \left(G^{MN} (D_M\Phi)^\dagger D_N\Phi - M^2|\Phi|^2\right) \\ \left(-\frac{1}{2}\mathrm{Tr}\,(W_{MN}W_{RS}) - \frac{1}{4}B_{MN}B_{RS}\right) G^{MR}G^{NS}\Big]$$

$$\Phi \sim (2, 1/2)$$

$$SU(2)_L \times U(1)_Y$$

$$\mathcal{S}_4 \;=\; \int \mathrm{d}^4x \int_{L_0}^{L_1} \mathrm{d}z\, \sqrt{G} \Big[\, \delta(z - L_0) G^{\mu\rho} G^{\nu\sigma} \\ \left[-\frac{1}{2} D \mathrm{Tr}\,[W_{\mu\nu}W_{\rho\sigma}] - \frac{1}{4} D B_{\mu\nu}B_{\rho\sigma}\right] \\ -\delta(z - L_i)\, 2\lambda_i \left(|\Phi|^2 - \frac{\mathrm{v}_i^2}{2}\right)^2$$

- Kinetic boundary terms needed for renormalization.
- Boundary terms introduce spontaneous EWSB.

# EWSB

- Bulk VEV for Higgs: $\langle\Phi\rangle = \frac{\mathrm{v}(z)}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$
- Bulk Equations: $\partial_z\left(\frac{L^3}{z^3}\partial_z \mathrm{v}\right) - \frac{L^5}{z^5}M^2\mathrm{v} = 0$
- Solution: $M^2 = -4/L^2$

$$\mathrm{v}(z) = Az^2 + Bz^2\log(z/L)$$

- Boundary terms: $\lambda_i \to +\infty$ $\quad \mathrm{v}(L_0) = \mathrm{v}_0\,, \quad \mathrm{v}(L_1) = \mathrm{v}_1\,,$

- Finally d=2: $\frac{\mathrm{v}_0}{L_0^2} = \frac{\mathrm{v}_1}{L_1^2}$

$$\mathrm{v}(z) = \frac{\mathrm{v}_1}{L_1^2}z^2 = \frac{\mathrm{v}_0}{L_0^2}z^2$$

# Electro-Weak Phenomenology

- Define:

$$V^M \equiv \frac{g' W_3^M + g B^M}{\sqrt{g^2 + g'^2}}$$

$$A^M \equiv \frac{g W_3^M - g' B^M}{\sqrt{g^2 + g'^2}}$$

- Bulk Equations:

$$A^\mu(q,z) \equiv A^\mu(q) v_Z(z,q)$$

$$\partial_z \frac{L}{z} \partial_z v_i - \mu_i^4 L z v_i = -q^2 \frac{L}{z} v_i$$

- Where:

$$\mu_W^4 = 1/4 g^2 \mathrm{v}_0^2 / L^2$$

$$\mu_Z^4 = 1/4 (g^2 + g'^2) \mathrm{v}_0^2 / L^2.$$

$$\mathcal{L} = \frac{P_{\mu\nu}}{2} A_i^{\mu} \pi_{ij}(q^2) A_j^{\nu} + g_4^a J_{a\mu} A_a^{\mu}$$

- Polarizations from UV-boundary Action

$$\begin{aligned}
\frac{\pi_+}{\mathcal{N}^2} &= Dq^2 + \frac{\partial_z v_W}{v_W}(q^2, L_0)\,, \\
\frac{\pi_{BB}}{\mathcal{N}^2} &= Dq^2 + \frac{g^2}{g^2 + g'^{\,2}} \frac{\partial_z v_v}{v_v}(q^2, L_0) + \frac{g'^{\,2}}{g^2 + g'^{\,2}} \frac{\partial_z v_Z}{v_Z}(q^2, L_0)\,, \\
\frac{\pi_{WB}}{\mathcal{N}^2} &= \frac{g g'}{g^2 + g'^{\,2}} \left( \frac{\partial_z v_v}{v_v}(q^2, L_0) - \frac{\partial_z v_Z}{v_Z}(q^2, L_0) \right)\,, \\
\frac{\pi_{WW}}{\mathcal{N}^2} &= Dq^2 + \frac{g'^{\,2}}{g^2 + g'^{\,2}} \frac{\partial_z v_v}{v_v}(q^2, L_0) + \frac{g^2}{g^2 + g'^{\,2}} \frac{\partial_z v_Z}{v_Z}(q^2, L_0)\,,
\end{aligned}$$

# Regularization

- Taking: $L_0 \longrightarrow L$
- Expanding for: $L_0 \longrightarrow 0$

$$\frac{\partial_z v_v}{v_v}(q^2, L_0) = q^2 L_0 \left( \frac{\pi}{2} \frac{Y_0(qL_1)}{J_0(qL_1)} - \left( \gamma_E + \ln \frac{qL_0}{2} \right) \right)$$

$$\frac{\partial_z v_Z}{v_Z}(q^2, L_0) = L_0 \left\{ \mu_Z^2 - q^2 \left[ \gamma_E + \ln(\mu_Z L_0) + \frac{1}{2}\psi\left(-\frac{q^2}{4\mu_Z^2}\right) - \frac{c_2}{2c_1}\Gamma\left(-\frac{q^2}{4\mu_Z^2}\right) \right] \right\}$$

- From Neumann at IR:

$$c_1 = 2L\left(-1 + \frac{q^2}{4\mu_Z^2}, \mu_Z^2 L_1^2\right) + L\left(\frac{q^2}{4\mu_Z^2}, -1, \mu_Z^2 L_1^2\right),$$

$$c_2 = -U\left(-\frac{q^2}{4\mu_Z^2}, 0, \mu_Z^2 L_1^2\right) + \frac{q^2}{2\mu_Z^2} U\left(1 - \frac{q^2}{4\mu_Z^2}, 1, \mu_Z^2 L_1^2\right)$$

# Renormalization

- Define, at finite UV cut-off:

$$D = L_0 \left( \ln \frac{L_0}{L_1} + \frac{1}{\varepsilon^2} \right)$$

$$\mathcal{N}^2 = \varepsilon^2 / L_0$$

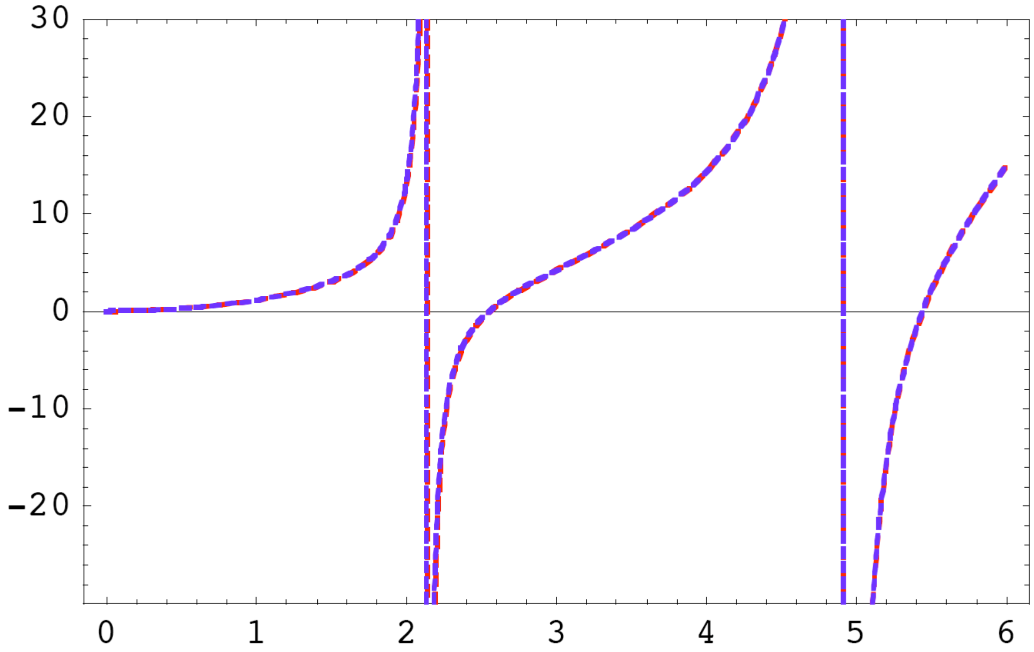
- Cut-off dependence disappears, take the limit UV cut-off -> Infinity. Gauge coupling kept fixed:

$$g_4^{(\prime)\,2} = \varepsilon^2 g^{(\prime)\,2} / L$$

# Polarizations

$$g/\sqrt{L} \sim 1.3$$

$$M_{\rho^0} \simeq 2.5 \text{ TeV}$$




$q = \sqrt{q^2}$ (TeV)

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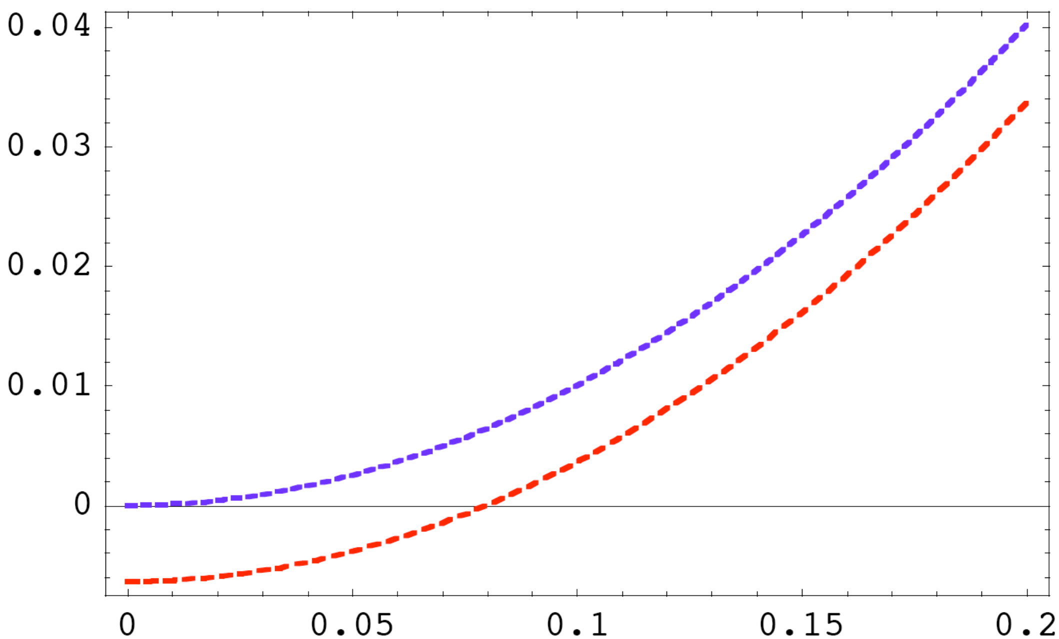


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# Phenomenology

- Assume: $\mu_Z^2 L_1^2 \ll 1$
- Spectrum: $M_{\rho^0} = k/L_1 \qquad k \in [2.4, 4.7]$

$$M_W^2 \simeq \varepsilon^2 \left( \mu_W^2 \tanh \frac{\mu_W^2 L_1^2}{2} \right) \simeq \frac{1}{2} \varepsilon^2 \mu_W^4 L_1^2 \, ;$$

$$M_Z^2 \simeq (g^2 + g'^2)/g^2 M_W^2$$

- EW precision observables:

$$\begin{aligned} \hat{T} &= \frac{\varepsilon^2}{M_W^2} \left( \mu_W^2 \tanh \frac{\mu_W^2 L_1^2}{2} - \frac{\mu_W^4}{\mu_Z^2} \tanh \frac{\mu_Z^2 L_1^2}{2} \right) \\ &\simeq \frac{\varepsilon^2}{M_W^2} \frac{\mu_W^4 L_1^6}{24} (\mu_Z^4 - \mu_W^4) \end{aligned}$$

$$\hat{S} = \varepsilon^2 \frac{1}{2e} \mu_W^4 L_1^4$$

# Experimental Bounds

- Experiment:

$$\hat{S}_{exp} = (-0.9 \pm 3.9) \times 10^{-3}\,,$$
$$\hat{T}_{exp} = (2.0 \pm 3.0) \times 10^{-3}\,,$$

- Theory:

$$\hat{S} \simeq \frac{1}{e} M_W^2 L_1^2 = \frac{k^2}{e} \frac{M_W^2}{M_{\rho^0}^2}\,,$$
$$\hat{T} = \frac{M_Z^2 - M_W^2}{6\varepsilon^2} L_1^2 = \frac{k^2}{6\varepsilon^2} \frac{M_Z^2 - M_W^2}{M_{\rho^0}^2}$$

- Bounds:

$$\frac{1}{L_1} > \frac{M_W}{\sqrt{e\hat{S}_{\max}}} = 890\,\mathrm{GeV}$$

- Techni-rho mass:

$$\varepsilon > 1/2 \;\; (g/\sqrt{L} < 1.3)$$
$$k(\varepsilon = 1/2) \simeq 2.8.$$

$$M_{\rho^0} \simeq 2.5 \;\; \mathrm{TeV}$$

# Fine-Tuning?

- Bounds evaded by: $\mu_Z^2 L_1^2 \ll 1$
- Look back at regularized theory:

$$M_W^2 = \frac{1}{8}\varepsilon^2 g^2 \mathrm{v}_1^2 \left(\frac{L_0}{L_1}\right)^2 = \frac{1}{4} g_4^2 \eta^2$$

$$\eta^2 = L\frac{\mathrm{v}_1^2}{2}\left(\frac{L_0}{L_1}\right)^2 = \frac{1}{\sqrt{2}G_F} \simeq (246\,\mathrm{GeV})^2$$

- Translation: $\frac{\mathrm{v}_1^2 L L_0^2}{2} = \eta^2 L_1^2 < \left(\frac{1}{3.6}\right)^2$

- Is it NATURAL?

# Some Estimates

- "Natural" value: $$v_1 \simeq \frac{2.4}{gL_1}$$
- From QCD… $$\sqrt{2} g_\rho f_\pi = M_\rho$$ $$g_\rho = g/\sqrt{L}$$
- ..and large q… $$L/g^2 = N_c/12\pi^2$$ $$g_\rho \simeq 6$$
- Conclusion: $$\frac{L}{g^2}\frac{L_0^2}{L_1^2} < \left(\frac{1}{6}\right)^2$$

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NEW NON-PERTURBATIVE

- Conclusion: $$\frac{L}{g^2}\frac{L_0^2}{L_1^2} < \left(\frac{1}{6}\right)^2$$

PERTURBATIVE RESULT

# More Estimates

- UV cut-off: $g/\sqrt{L} \sim 1.3$

$$1/L_0 \sim 6/L_1 \sim 5.3 \text{ TeV}$$

- Localized Top: $-\delta(z-L_0) y_u \bar{q}_L \tilde{\Phi} u_R$

$$\frac{y_u}{\sqrt{L}} = \frac{L_1}{\sqrt{2} L_0}$$

$$y_u/\sqrt{L} \sim 4$$

- Perturbative: $N_d \sim 2N_T \qquad N_T \sim 8.$

$$\hat{S}_p = \frac{\alpha}{4\sin^2\theta_W} \frac{N_d N_T}{6\pi} \sim 0.06 \qquad \text{vs.} \qquad \hat{S} \simeq 0.003$$

# Systematic Errors

- Large N: 5% ?
- Model Dependences: 50% ??
- Departure from ADS5: 50% ??
- Higher order operators: 50% ??

vs.

- Perturbative Estimate: 2000% !!!!!!!

# What's next?

- LHC-phenomenology: production cross-sections and decay rates.
- LHC-phenomenology: where is the Higgs?
- Fine-tuning study: stabilization a` la GW?.
- Fermion model-building: hierarchies in mass? CKM? FCNC?
- Generalizations: are T and S always positive? Is there a simple formula for general d? What about departures from AdS5?

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